

### 3.1 Acute Angle Triangles - Key

## Acute Angle Triangles [3.1]

## Trigonometry

Trigonometry means “three angle measurement”. It is a branch of mathematics which deals with the measurement of angles and sides of triangles.

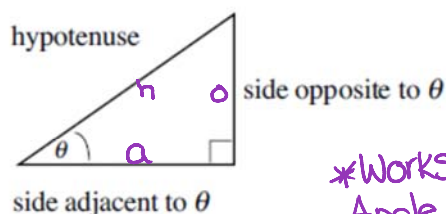
## Primary Trigonometric Ratios

Complete the following

$$\text{sine} \Rightarrow \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{cosine} \Rightarrow \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{tangent} \Rightarrow \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$



\*Works for Right Angle Triangles\*

These ratios are called the *Primary Trigonometric Ratios* and can be remembered by the acronym **SOHCAHTOA**.

\*Always check to see your calculator is in Degrees!\*

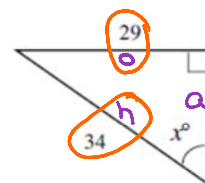


Use the diagram to determine

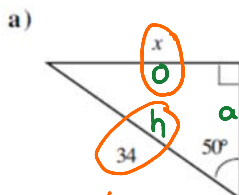
$$\text{a) } \sin x^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{29}{34} = 0.8529$$

$$\text{b) } x^\circ = \sin^{-1}(0.8529)$$

$$x = 58.5^\circ$$



In each diagram find the length of the indicated side to the nearest tenth.



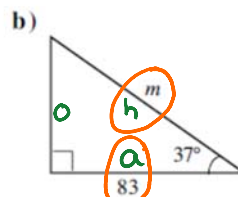
$$\sin 50^\circ = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 50^\circ = \frac{x}{34}$$

$$x = 34 (\sin 50^\circ)$$

$$x = 26.0$$

→ I need x alone



$$\cos 37^\circ = \frac{\text{adj}}{\text{hyp}}$$

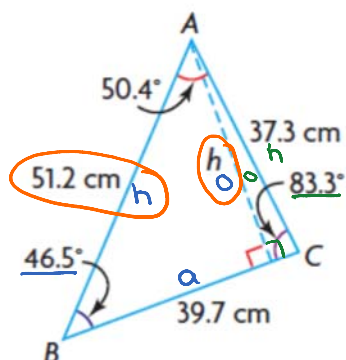
$$\cos 37^\circ = \frac{83}{m}$$

$$\frac{m(\cos 37^\circ)}{\cos 37^\circ} = \frac{83}{\cos 37^\circ}$$

$$m = \frac{83}{\cos 37^\circ}$$

$$m = 103.9$$

Ex: What are two equivalent expressions that represent the height of  $\triangle ABC$ ?

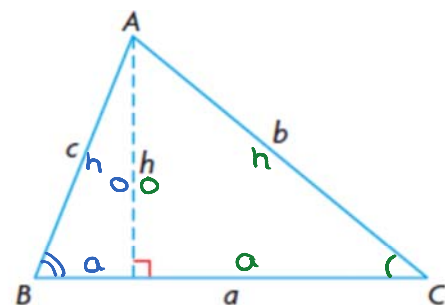


Option 1  
 $\sin 46.5^\circ = \frac{h}{51.2}$

$$h = 51.2 (\sin 46.5^\circ)$$

$$h = 37.1$$

Option 2  
 $\sin 83.3^\circ = \frac{h}{37.3}$   
 $h = 37.3 (\sin 83.3^\circ)$   
 $h = 37.0$



$$\sin C = \frac{\text{opp}}{\text{hyp}}$$

$$\sin B = \frac{\text{opp}}{\text{hyp}}$$

$$\sin C = \frac{h}{b}$$

$$\sin B = \frac{h}{c}$$

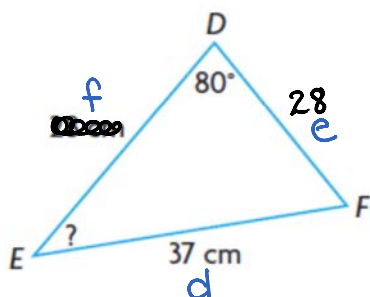
$$b(\sin C) = h$$

$$c(\sin B) = h$$

$$\frac{b(\sin C)}{\sin C} = \frac{c(\sin B)}{\sin C}$$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Ex: Explain how you could determine the measure of  $\angle E$  in this acute triangle



No 90°  $\therefore$  No SOHCAHTOA  
 We need 3 of 4 'clues' to solve the puzzle.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{d}{\sin D} = \frac{e}{\sin E}$$

$$\frac{37}{\sin 80^\circ} = \frac{28}{\sin E}$$

\*Cross Mult\*

$$\frac{37(\sin E)}{37} = \frac{28(\sin 80^\circ)}{37}$$

$$\sin E = \frac{28(\sin 80^\circ)}{37}$$

$$E = \sin^{-1}\left(\frac{28(\sin 80^\circ)}{37}\right)$$

$$E = 48^\circ$$

\*In groups of 2/3 complete the "Getting Started Activity on page 114 on your Text.

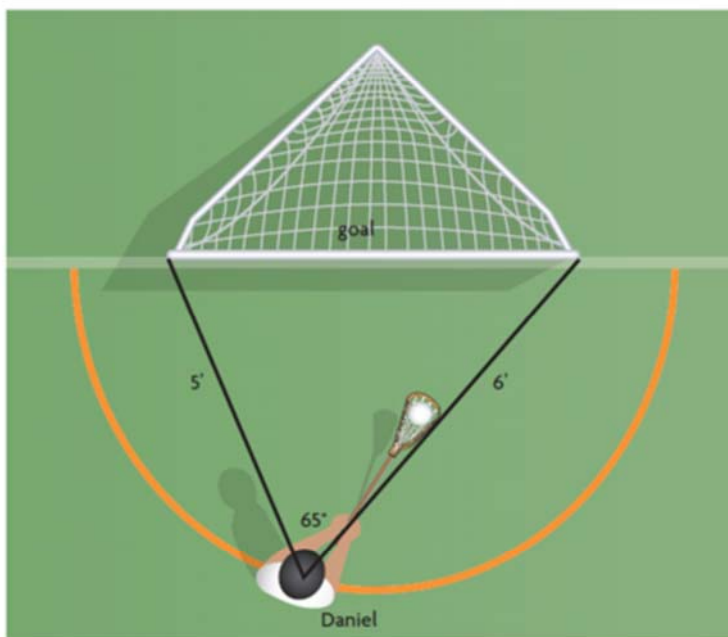
Hand in your estimates as you leave ☺

### 3.1 Lacrosse Net - Key



**Getting Started Activity: The Lacrosse Net [3.1]**

\* Read through the entire activity on p. 114 before getting started in your small group of 2 or 3 \*



Daniel is about to take a shot at a field lacrosse net. He estimates his current position, as above.

- Does Daniel's position form a right triangle with the goalpost?
- A primary trigonometric ratio cannot be used to determine the width of the net directly. Explain why.
- Copy the triangle that includes Daniel's position in the diagram above. Add a line segment so that you can determine a height of the triangle using trigonometry.
- Determine the height of the triangle using a primary trig ratio.

e) Create a plan that will allow you to determine the width of the lacrosse net using the two right triangles you created.

f) Carry out your plan to determine the width of the net.

### 3.2 Sine Law - Key

## Sine Law [3.2]

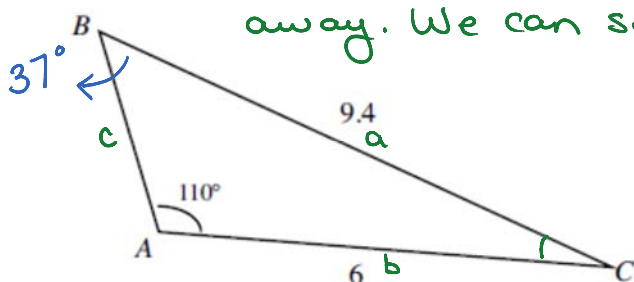
In every triangle  $ABC$ ,  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$  or  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

↗ when finding a side

↖ when finding an angle

Use the sine law in the triangle shown below to determine the measure of  $\angle ACB$  to the nearest degree.

We don't have enough information to solve for  $\angle C$  right away. We can solve for  $\angle B$ , so we'll start with that.



$$\textcircled{1} \frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{6} = \frac{\sin 110^\circ}{9.4}$$

$$\sin B = \frac{6(\sin 110^\circ)}{9.4}$$

$$\sin B = 0.5998\dots$$

$$\angle B = \sin^{-1}(0.5998)$$

$$\underline{\angle B = 37^\circ}$$

$\textcircled{2}$  Solve for  $\angle C$ :

$$\Delta s = 180^\circ$$

$$\begin{array}{r} 180^\circ \\ - 110^\circ \\ - 37^\circ \\ \hline \end{array}$$

$$\boxed{33^\circ}$$

A surveyor measures a base line  $PQ$  440m long. He takes measurements of a landmark  $R$  from  $P$  and  $Q$ , and finds that  $\angle QPR = 46^\circ$  and  $\angle PQR = 75^\circ$ .

a) Calculate the perimeter of  $\triangle PQR$  to the nearest metre.

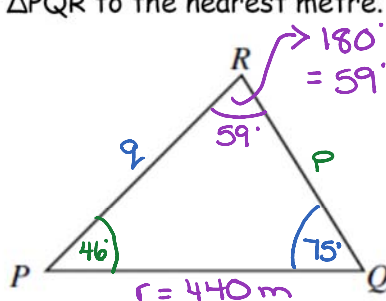
Side PR

$$\frac{p}{\sin Q} = \frac{r}{\sin R}$$

$$\frac{p}{\sin 75^\circ} = \frac{440}{\sin 59^\circ}$$

$$p = \frac{440(\sin 75^\circ)}{\sin 59^\circ}$$

$$\underline{\underline{p = 495.85 \text{ m}}}$$



Side RQ

$$\frac{p}{\sin P} = \frac{r}{\sin R}$$

$$\frac{p}{\sin 46^\circ} = \frac{440}{\sin 59^\circ}$$

$$p = \frac{440(\sin 46^\circ)}{\sin 59^\circ}$$

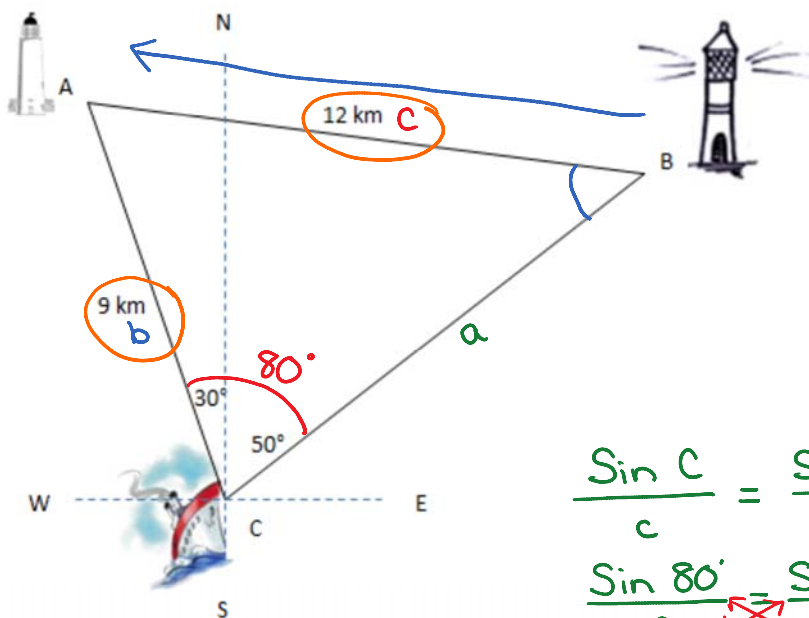
$$\underline{\underline{p = 369.25}}$$

Perimeter

$$495.85 + 369.25 + 440$$

$$\underline{\underline{1305 \text{ m}}}$$

The captain of a small boat is delivering supplies to two lighthouses, as shown. His compass indicates that the lighthouse to his left is located at  $N30^\circ W$  and the lighthouse to his right is located at  $N50^\circ E$ . determine the compass direction he must follow when he leaves lighthouse B for lighthouse A.



$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin 80^\circ}{12} = \frac{\sin B}{9}$$

$$\frac{9(\sin 80^\circ)}{12} = \frac{12(\sin B)}{12}$$

$$\sin B = \frac{9(\sin 80^\circ)}{12}$$

$$\angle B = \sin^{-1}\left(\frac{9(\sin 80^\circ)}{12}\right)$$

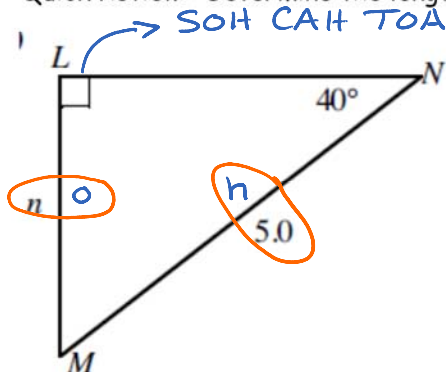
$$\angle B = 47.6$$

$$\boxed{\angle B = 48^\circ}$$

### 3.3 Cosine Law - Key

## Cosine Law [3.3]

Quick Review: Determine the length of the indicated side to the nearest tenth.



$$\sin 40^\circ = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 40^\circ = \frac{n}{5.0}$$


$$n = 5.0 (\sin 40^\circ)$$

$$\boxed{n = 3.2}$$



To solve for an angle or a side in a triangle using trigonometry, three pieces of information are required, one of which must be the length of a side.

Use the following guide to determine which law to use in any given situation:

- In a right-angled triangle use SOHCAHTOA 
- Use Law of Cosines if you are given either;
  - i) all three sides, or
  - ii) two sides and the contained angle.
- In all other cases use the Law of Sines.



The equation

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$\text{OR } c^2 = a^2 + b^2 - 2ab \cos C$$

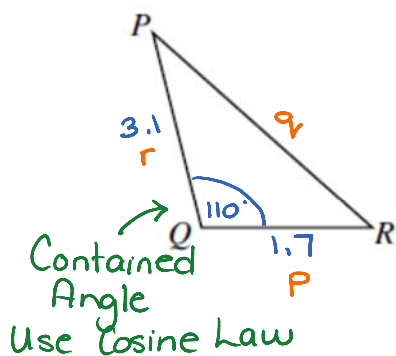
can be rearranged to the form

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

This form of the cosine law can be used to determine any angle in a triangle when we are given the length of all three sides, (SSS)

We use cosine law to determine an unknown side length or angle measure in an acute triangle

Find the length, to the nearest tenth of a cm, of the third side of  $\triangle PQR$  if  $QP = 1.7$  cm,  $QR = 1.7$  cm,  $QR = 3.1$  cm and  $\angle PQR = 110^\circ$ .



$$q^2 = r^2 + p^2 - 2rp \cos Q$$

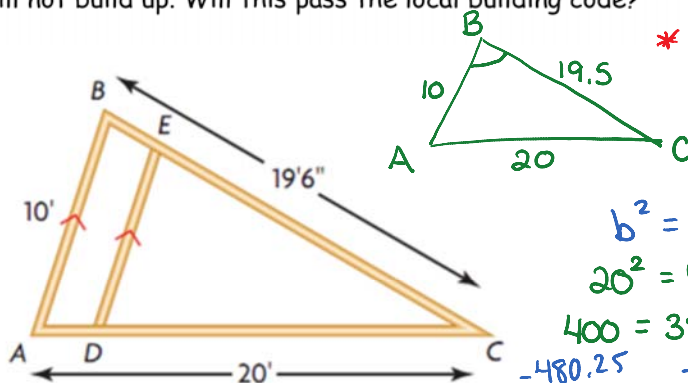
$$q^2 = (1.7)^2 + (3.1)^2 - 2(1.7)(3.1) \cos 110^\circ$$

$$q^2 = 16.104 \dots$$

$$q = \sqrt{16.104}$$

$$q = 4.0 \text{ cm}$$

The diagram shows the plan for a roof, with support beam  $DE$  parallel to  $AB$ . The local building code requires the angle formed at the peak of a roof to fall within a range of  $70^\circ$  to  $80^\circ$  so that snow and ice will not build up. Will this pass the local building code?



\* All 3 sides > Use Cosine Law!

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$20^2 = (19.5)^2 + (10)^2 - 2(19.5)(10) \cos B$$

$$400 = 380.25 + 100 - 390 \cos B$$

$$-480.25 = -390 \cos B$$

$$\frac{-80.25}{-390} = \frac{-390 \cos B}{-390}$$

$$\cos B = \frac{80.25}{390}$$

$$B = \cos^{-1}\left(\frac{80.25}{390}\right)$$

$$\angle B = 78^\circ$$

So yes! It will pass

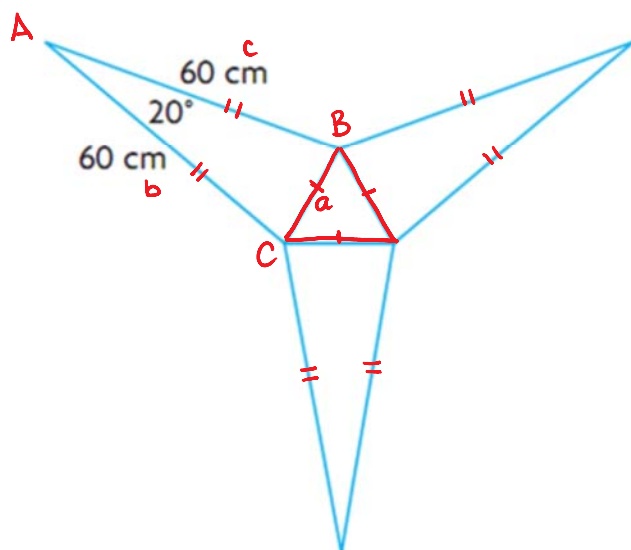
\* Two Column Quiz tomorrow → 1 Sine Law / 1 Cosine Law



### 3.4 (1) Problem Solving - Key

### Problems Involving Acute Triangles - Part 1 [3.4]

**Warming up:** A three-pointed star is made up of an equilateral triangle and three congruent isosceles triangles. Determine the length of each side of the equilateral triangle in this three-pointed star. Round the length to the nearest centimetre.



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 60^2 + 60^2 - 2(60)(60) \cos 20^\circ$$

$$a^2 = 3600 + 3600 - 7200 \cos 20^\circ$$

$$a^2 = 7200 - 7200 \cos 20^\circ$$

$$a^2 = 434.213$$

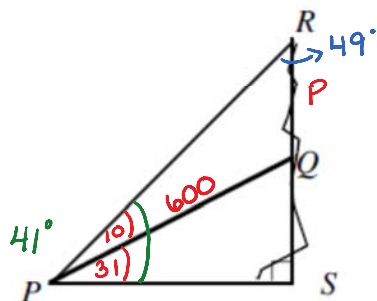
$$a = 21 \text{ cm}$$

### How do we know when to use what?

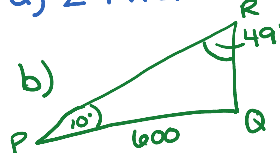
Clues	2 sides & opposite one of the sides	2 Angles and a side	2 sides and the contained angle	3 sides (SSS)
Quick Sketch				
Looking for >	angle	side	side	Angle
Use >>	Sine Law	Sine Law	Cosine Law	Cosine Law

**Example 1:** P and Q are two bases for a mountain climb. PQ is 600m and QR is a vertical stretch of a rock face. The angle of elevation of Q from P is  $31^\circ$ , and the angle of elevation of R from P is  $41^\circ$ .

- Mark these measurements on the diagram and state the measure of  $\angle PRQ$ .
- Use sine law in  $\triangle PQR$  to calculate the height of the vertical climb, QR, to the nearest metre.



$$a) \angle PRQ = \angle PRS = 180^\circ - 90^\circ - 41^\circ = 49^\circ$$



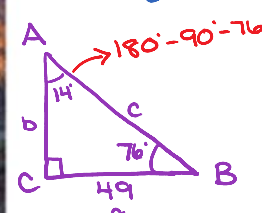
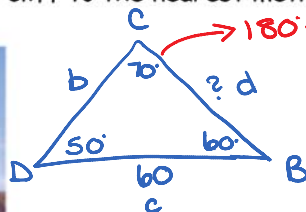
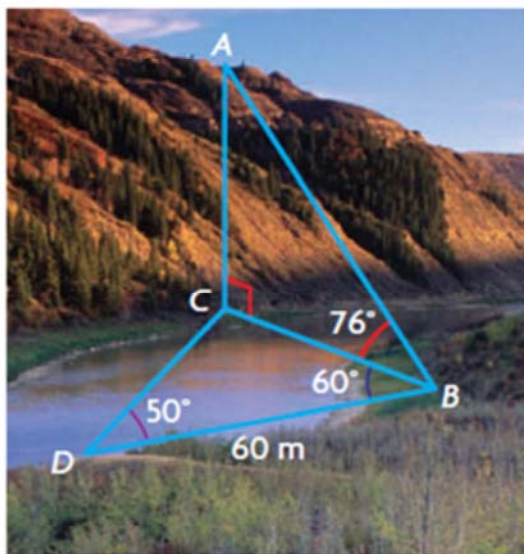
$$\frac{P}{\sin P} = \frac{r}{\sin R}$$

$$\frac{P}{\sin 10^\circ} = \frac{600}{\sin 49^\circ}$$

$$P = \frac{600 (\sin 10^\circ)}{\sin 49^\circ}$$

$$P = 138 \text{ m}$$

**Example 2:** Brendan & Diana plan to climb the cliff at Dry Island Buffalo Jump, Alberta. They need to know the height of the climb before they start. Brendan stands at point B, as shown in the diagram. He uses a clinometer to determine  $\angle ABC$ , the angle between the base of the cliff, himself, and Diana, who is standing at point D. Diana estimates  $\angle CDB$ , the angle between the base of the cliff, herself, and Brendan. Determine the height of the cliff to the nearest metre.



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{49}{\sin 14^\circ} = \frac{b}{\sin 76^\circ}$$

$$b(\sin 14^\circ) = 49(\sin 76^\circ)$$

$$b = \frac{49(\sin 76^\circ)}{\sin 14^\circ} \rightarrow 197 \text{ m}$$

$$\frac{c}{\sin C} = \frac{d}{\sin D}$$

$$\frac{60}{\sin 70^\circ} = \frac{d}{\sin 50^\circ}$$

$$\frac{d(\sin 70^\circ)}{\sin 70^\circ} = \frac{60(\sin 50^\circ)}{\sin 70^\circ}$$

$$d = \frac{60(\sin 50^\circ)}{\sin 70^\circ}$$

$$d = 49 \text{ m}$$

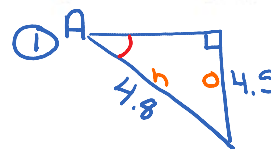
### 3.4 (2) Problem Solving - Key

#### Problems Involving Acute Triangles - Part 2 [3.4]

**Example 1:** Determine the angle of depression, to the nearest degree, for each camera.



Two security cameras in a museum must be adjusted to monitor a new display of fossils. The cameras are mounted 6 m above the floor, directly across from each other on opposite walls. The walls are 12 m apart. The fossils are displayed in cases made of wood and glass. The top of the display is 1.5 m above the floor. The distance from the camera on the left to the centre of the top of the display is 4.8 m. Both cameras must aim at the centre of the top of the display.

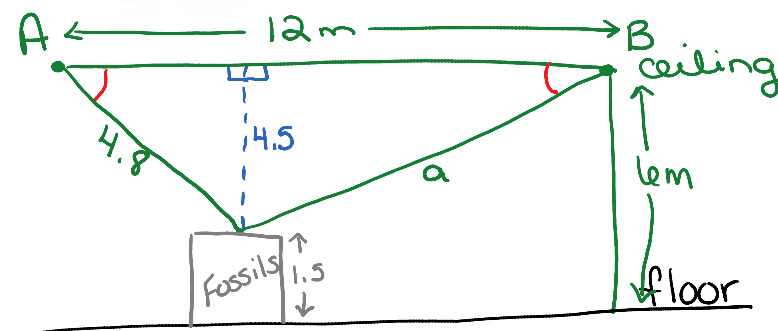


$$\sin A = \frac{\text{opp}}{\text{hyp}}$$

$$\sin A = \frac{4.5}{4.8}$$

$$A = \sin^{-1}(4.5/4.8)$$

$$\angle A = 70^\circ$$



② Find side a:



$$a^2 = b^2 + c^2 - 2ac \cos A$$

$$a^2 = 4.8^2 + 12^2 - 2(4.8)(12) \cos 70^\circ$$

$$a^2 = 127.639 \dots$$

$$a = 11.3 \text{ m}$$

③ Find  $\angle B$ :



$$\sin B = \frac{\text{opp}}{\text{hyp}}$$

$$\sin B = \frac{4.5}{11.3}$$

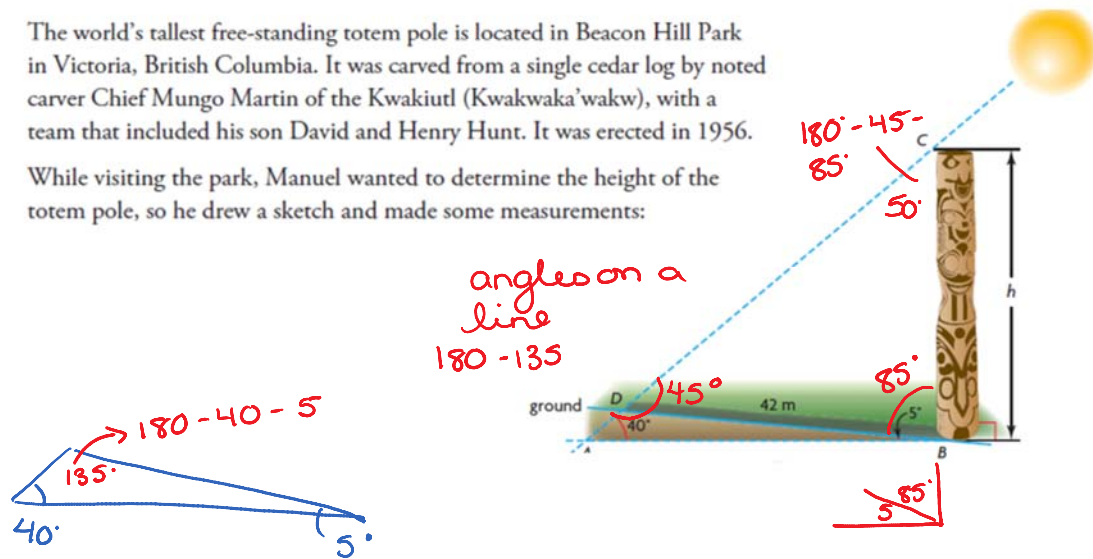
$$B = \sin^{-1}\left(\frac{4.5}{11.3}\right)$$

$$B = 23^\circ$$

**Example 2:**

The world's tallest free-standing totem pole is located in Beacon Hill Park in Victoria, British Columbia. It was carved from a single cedar log by noted carver Chief Mungo Martin of the Kwakiutl (Kwakwaka'wakw), with a team that included his son David and Henry Hunt. It was erected in 1956.

While visiting the park, Manuel wanted to determine the height of the totem pole, so he drew a sketch and made some measurements:



- I walked along the shadow of the totem pole and counted 42 paces, estimating each pace was about 1 m.
- I estimated that the **angle of elevation** of the Sun was about  $40^\circ$ .
- I observed that the shadow ran uphill, and I estimated that the angle the hill made with the horizontal was about  $5^\circ$ .

How can Manuel determine the height of the totem pole to the nearest metre?

$$\frac{c}{\sin C} = \frac{d}{\sin D}$$

$$\frac{42}{\sin 50^\circ} = \frac{d}{\sin 45^\circ}$$

$$\frac{d(\sin 50^\circ)}{\cancel{\sin 50^\circ}} = \frac{42(\sin 45^\circ)}{\sin 50^\circ}$$

$$\boxed{d = 39\text{m}}$$